

Research Center for their interest, availability of measurements, and valuable discussions throughout the project.

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An Improved Model for the Dielectric Constant of Sea Water at Microwave Frequencies

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Abstract—The advent of precision microwave radiometry has placed a stringent requirement on the accuracy with which the dielectric constant of sea water must be known. To this end, measurements of the dielectric constant have been conducted at *S*-band and *L*-band with a quoted uncertainty of tenths of a percent. These and earlier results are critically examined, and expressions are developed which will yield computations of brightness temperature having an error of no more than 0.3 K for an undisturbed sea at frequencies lower than *X*-band. At the higher microwave and millimeter wave frequencies, the accuracy is in question because of uncertainties in the relaxation time and the dielectric constant at infinite frequency.

INTRODUCTION

DURING the late forties and early fifties, several experiments were conducted to define the complex dielectric constant of saline solutions as a function of temperature and salinity at various frequencies within the microwave band [1]–[5]. Renewed interest has developed in recent years due to the introduction of microwave radiometers as possible remote sensors of temperature and salinity of the ocean and estuarial waters [6]–[8]. The connection between the microwave noise power received by a radiometer and the dielectric constant is the emissivity, or equivalently, the power absorption coefficient of the water. For a body of water that fills a flat half-space, the absorption coefficient is defined through the usual Fresnel equations, and a mathematical relationship between the emissivity and the dielectric constant is thereby established.

In 1971, Stogryn [8] assimilated the best of the early experimental work and derived regression fits to the dielectric parameters. These equations have been convenient for the researcher to use; however, the problem with Stogryn's fit is that the quoted accuracy of the experimental data [1]–[5] is insufficient to match the measurement capability that current radiometers can provide. This accuracy problem was recognized during the study phase of the development of a precision *S*-band radiometer that was designed to measure brightness temperature to within absolute and relative accuracies of 0.3 K and 0.1 K, respectively [9]. A program was, therefore, initiated to carefully measure the dielectric constant of sea water at 2.65 GHz over a range of temperatures and salinities typical for the oceans of the world. Subsequent to the successful development of the *S*-band instrument, a companion *L*-band radiometer was constructed and the dielectric constant measurements were repeated for a wider range of salinities at 1.43 GHz [11]. It is, therefore, the purpose of this paper to reexamine the Stogryn regression fit, utilizing early high frequency microwave data, and the more recent *S*- and *L*-band results. It is believed that the regression fit reported herein will provide a brightness temperature calculation accurate to within 0.3 K at frequencies below *X*-band.

REVIEW OF RECENT EXPERIMENTAL MEASUREMENTS

Measurements of the dielectric constant of saline solutions at 2.65 GHz have recently been reported by Ho and his co-workers [10]–[11]. A perturbation technique was used, whereby sample tubes of small but known volume were

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TABLE I
VALUES FOR LINEAR FIT COEFFICIENTS AS FUNCTION OF TEMPERATURE
($f = 1.43$ GHz) [11]

Temp (°C)	5	10	20	30
A_0	1.0040	1.0015	1.0017	1.0016
A_1	5.67×10^{-3}	5.56×10^{-3}	5.50×10^{-3}	5.14×10^{-3}
C_0	0.1368	0.1178	0.0864	0.0653
C_1	0.02765	0.03282	0.04391	0.05809

TABLE II
VALUES FOR LINEAR FIT COEFFICIENTS AS FUNCTION OF TEMPERATURE
($f = 2.653$ GHz) [10]

Temp (°C)	5.5	15	24	30
A_0	1.0000	1.0000	1.0000	1.0000
A_1	6.017×10^{-3}	6.106×10^{-3}	6.115×10^{-3}	6.080×10^{-3}
C_0	0.2519	0.1750	0.1333	0.1150
C_1	0.01557	0.02160	0.02797	0.03100

inserted along the axis of a cylindrical cavity excited in the TM_{010} mode. Values of the complex dielectric constant of the sample were then derived from the resultant shift in the resonant frequency and the change in the quality factor Q of the cavity. A straightforward perturbation analysis shows that the shift in the resonant frequency δ is proportional to $\epsilon' - 1$, where ϵ' is the real part of the dielectric constant of the sample. Furthermore, if the frequency shifts produced by two samples a and b are measured, the analysis yields [10]

$$\frac{\epsilon'_a - 1}{\epsilon'_b - 1} = \frac{\delta_a}{\delta_b} \quad (1)$$

This is an interesting relationship because if ϵ'_a is chosen as a reference, only one absolute measurement of ϵ' is required. Subsequent measurements of arbitrary samples are done on a relative basis; consequently sources of systematic errors tend to cancel out. Distilled water was the natural choice for the reference because of the wealth of available experimental data [3]–[5].

The imaginary part of the dielectric constant ϵ'' of a sample is derived from measurements of the change of the inverse Q of the cavity. Indeed, the specific relationship, as derived from perturbation theory, is [10]

$$\frac{\epsilon''}{\epsilon' - 1} = \frac{f_0}{2\delta[\Delta(1/Q)]} \quad (2)$$

where f_0 is the resonant frequency and the subscript b has been suppressed from ϵ'' and ϵ' . The sea water samples for the S -band measurements were obtained from various parts of the world, with the resulting salinities ranging from 29.6 to 35.4 parts per thousand (‰). The specific temperatures

selected for the measurements were 5.5, 15, 24, and 30°C. Dielectric constant measurements of NaCl solutions were also conducted at the first three of these temperatures with the normality ranging from 0.3N to 0.7N.

As part of a related microwave radiometer program, additional measurements were conducted at a frequency of 1.43 GHz [11]. Samples were collected from the Chesapeake Bay and the Pacific Ocean with the salinity ranging from 4.14 to 35.25‰. Measurements of the dielectric constant of these samples were conducted at temperatures of 5, 10, 20, and 30°C.

Both sets of measurements suggested that (1) and (2) vary linearly with chlorinity (chlorinity = (salinity - 0.03)/1.805 in sea water) [12] for a given temperature such that

$$\frac{\epsilon'_a - 1}{\epsilon' - 1} = A_0 + A_1 X \quad (3)$$

$$\frac{\epsilon''}{\epsilon' - 1} = C_0 + C_1 X \quad (4)$$

where X is the chlorinity in parts per thousand. Ho notes that the scatter of the data about the linear fit is less than 0.3 percent for (3) and 0.4 percent for (4) [10]. The numerical values of the four coefficients are listed in Tables I and II for 1.43 GHz and 2.653 GHz, respectively. Note that the L -band coefficient A_0 is not quite unity; consequently, the value of ϵ' will not reduce to the distilled water result as $X \rightarrow 0$. Ho asserts that this may be due to the presence of sulphates and nitrates in the estuarial waters sampled.

The results of absolute measurements of ϵ' and ϵ'' for distilled water are given in Tables III and IV for 1.43 GHz and 2.653 GHz, respectively. These data provide the normalization required to derive the absolute values of ϵ' and ϵ'' for the sea water samples.

TABLE III
 ϵ' AND ϵ'' FOR DISTILLED WATER AS FUNCTION OF TEMPERATURE
 $(f = 1.43 \text{ GHz})$ [11]

Temperature (°C)	ϵ'	ϵ''
5	84.54	10.742
10	82.96	8.888
20	79.56	6.285
30	76.14	4.783

TABLE IV
 ϵ' AND ϵ'' FOR DISTILLED WATER AS FUNCTION OF TEMPERATURE
 $(f = 2.653 \text{ GHz})$ [10]

Temperature (°C)	ϵ'	ϵ''
5.5	80.52	20.03
15	79.57	13.75
24	77.44	10.18
30	75.88	8.61

INTERPRETATION OF DATA

The complex dielectric constant of sea water may be calculated at any frequency within the microwave band from the Debye expression which, in its most general form, is given by [13]

$$\epsilon = \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1 + (j\omega\tau)^{1-\alpha}} - j \frac{\sigma}{\omega\epsilon_0} \quad (5)$$

where $\omega = 2\pi f$ is the radian frequency with f in hertz, ϵ_{∞} is the dielectric constant at infinite frequency, ϵ_s is the static dielectric constant, τ is the relaxation time in seconds, σ is the ionic conductivity in mhos/meter, α is an empirical parameter that describes the distribution of relaxation times, and $\epsilon_0 = 8.854 \times 10^{-12}$ is the permittivity of free space in farads/meter. The simplicity of the Debye expression is deceptive because ϵ_s , τ , and σ are all functions of the temperature and salinity of sea water.

In order for (5) to be of practical computational value to the user, the specific dependence must be derived from experimental results utilizing regression fits to the data. The variation of σ as a function of temperature and salinity can be deduced through relatively simple experimental techniques [12], [14], [15]. The measurement of ϵ_s would seem to favor very low frequency methods; however, there is evidence that the static dielectric constant for distilled water is slightly higher at microwave frequencies than at kilohertz frequencies [16]. Concerning the other parameters, the scale of the relaxation time is of such magnitude that meaningful values of ϵ_{∞} , α , and τ are best derived from microwave measurements. Measurements of ϵ_s and τ for NaCl solutions have been tabulated in a number of papers with quoted accuracies of approximately ± 3 percent for ϵ_s and ± 4 percent for τ [1]–[5]. Stogryn points out that there is no evidence showing that ϵ_{∞} depends on salinity and that the two data sets which address a possible temperature dependency conflict with each other. He finds that the

best fit to available high frequency data is achieved if ϵ_{∞} is assumed a constant value of 4.9. The estimated experimental uncertainty in ϵ_{∞} is about ± 20 percent. The experimental data of Grant *et al.* indicate that $\alpha = 0.02 \pm 0.007$ [5].

In order to critically evaluate Ho's measurements and to relate them to prior work, (5) was put into the form $\epsilon = \epsilon' - j\epsilon''$ and then rearranged to give the following expressions for ϵ_s and τ :

$$\epsilon_s = \epsilon_{\infty} + (\epsilon' - \epsilon_{\infty}) \left\{ 1 + \frac{(\epsilon'' - \sigma/\omega\epsilon_0) \sin \frac{\alpha\pi}{2}}{(\epsilon' - \epsilon_{\infty}) \cos \frac{\alpha\pi}{2}} + \frac{(\epsilon'' - \sigma/\omega\epsilon_0)^2}{(\epsilon' - \epsilon_{\infty}) \left[(\epsilon' - \epsilon_{\infty}) \cos \frac{\alpha\pi}{2} - (\epsilon'' - \sigma/\omega\epsilon_0) \sin \frac{\alpha\pi}{2} \right] \cos \frac{\alpha\pi}{2}} \right\} \quad (6)$$

$$\tau = \frac{1}{\omega} \left[\frac{\epsilon'' - \sigma/\omega\epsilon_0}{(\epsilon' - \epsilon_{\infty}) \cos \frac{\alpha\pi}{2} - (\epsilon'' - \sigma/\omega\epsilon_0) \sin \frac{\alpha\pi}{2}} \right]^{1/(1-\alpha)} \quad (7)$$

The distilled water values of ϵ' and ϵ'' listed in Tables III and IV were substituted into (6) in order to calculate ϵ_s at the various temperatures for comparison with published results. For distilled water, it was assumed that $\sigma = 0$ and $\epsilon_{\infty} = 4.9$; the parameter α was assigned a value of zero for a first-order iteration. The inverted data for ϵ_s are shown as a function of temperature in Fig. 1. The solid curve of ϵ_s versus temperature is a plot of Stogryn's regression equation based on a fit to the low frequency data of Malmberg and Maryott [4]. The dashed curve in figure 1 represents the results of Hasted's constraint fitting program as applied to all available experimental results conducted at the microwave frequencies [16]. The data points are in excellent agreement with Hasted's results.

In order to assess the influence of the distribution parameter α , the data points were rederived for $\alpha = 0.03$, which represents an upper bound to the experimental value quoted by Grant *et al.* This resulted in an upward shift in the data by as much as one unit of ϵ_s above Hasted's curve. It was, therefore, concluded that the standard Debye equation, with $\alpha = 0$, provides a sufficiently accurate model for the static dielectric constant at the lower microwave frequencies.

Based on the above considerations, the regression fit for the static dielectric constant of distilled water for $\alpha = 0$, as generated from the L- and S-band data, is given by

$$\epsilon_s^{\text{DW}}(T) = 88.045 - 0.4147T + 6.295 \times 10^{-4}T^2 + 1.075 \times 10^{-5}T^3 \quad (8)$$

where T is the temperature in degrees centigrade.

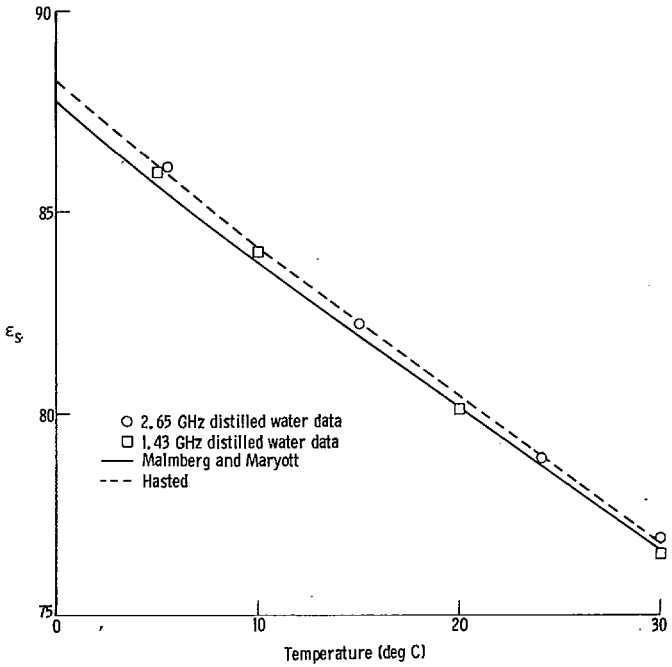


Fig. 1. Static dielectric constant of distilled water as function of temperature.

The process of evaluating the various Debye parameters for sea water tacitly assumes that the error resulting from uncertainties in the ionic conductivity of sea water is negligible. Cox quotes an error of ± 0.03 percent for salinities between 30 and 40‰, but does not present an error estimate for lower values of salinity due to uncertainties in the properties of the diluting water [12]. The low percentage of error at the high concentrations does, however, suggest that the conductivity is known to within the target accuracy of Ho's measurements.

The regression equation for the ionic conductivity of sea water, as derived by Weyl [15] and modified by Stogryn, is defined through the following relationships:

$$\sigma(T, S) = \sigma(25, S) \exp(-\Delta\beta) \quad (9)$$

$$\Delta = 25 - T \quad (10)$$

$$\begin{aligned} \beta = & 2.033 \times 10^{-2} + 1.266 \times 10^{-4}\Delta \\ & + 2.464 \times 10^{-6}\Delta^2 - S(1.849 \times 10^{-5} \\ & - 2.551 \times 10^{-7}\Delta + 2.551 \times 10^{-8}\Delta^2) \end{aligned} \quad (11)$$

and

$$\begin{aligned} \sigma(25, S) = & S(0.182521 - 1.46192 \times 10^{-3}S \\ & + 2.09324 \times 10^{-5}S^2 - 1.28205 \times 10^{-7}S^3) \end{aligned} \quad (12)$$

where T is temperature in degrees centigrade and S is salinity in parts per thousand. The above equations were used to subtract $\sigma/\omega\epsilon_0$ from ϵ'' in order to invert for ϵ_s and τ . As before, ϵ_∞ and α were assigned constant values of 4.9 and 0.0, respectively.

The L -band data were exclusively chosen to establish the polynomial fit for ϵ_s because the range of salinities was more

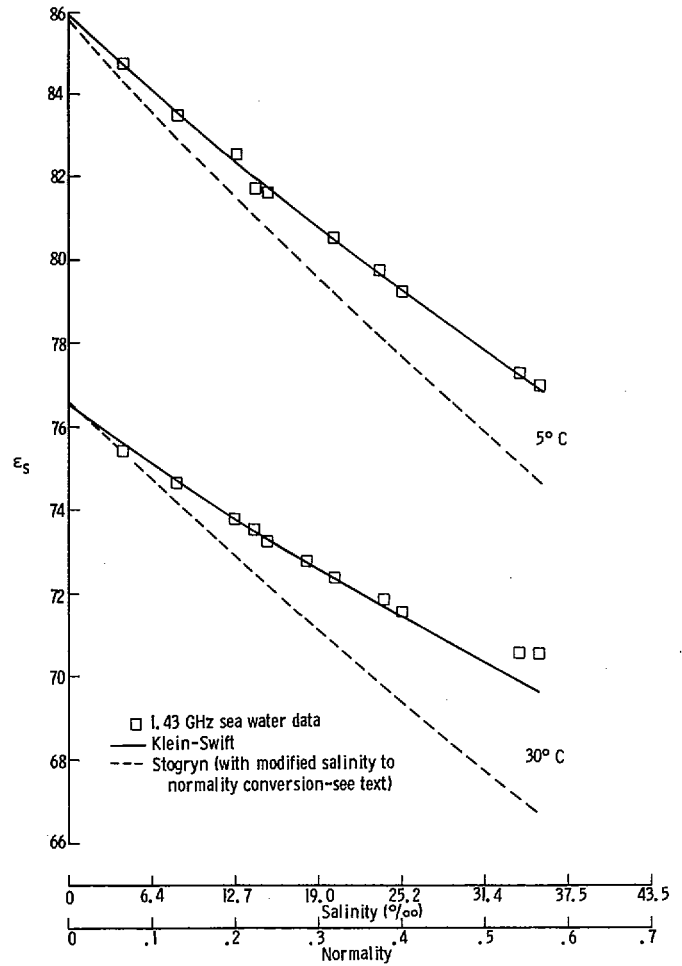


Fig. 2. Static dielectric constant of sea water as function of normality for temperatures of 5 and 30°C.

extensive. The S -band data were, however, used to check the fit for consistency. Instead of inverting ϵ' and ϵ'' as found from the published linear fit coefficients given in Tables I and II, actual raw data values of ϵ' and ϵ'' were kindly provided by Ho as a means of determining the fit and then checking the data scatter about it. The inverted data for ϵ_s are given as a function of salinity for 5 and 30°C in Fig. 2 and for 10 and 20°C in Fig. 3. The solid curves represent our polynomial fit through the data and the dashed curves are the corresponding values of ϵ_s as derived from Stogryn's equations, where the salinity into normality conversion has been modified as described later. Excluding the three high salinity points at 30°C as experimental error, the maximum scatter of the input data points is no more than 0.38 while the rms scatter is 0.21 units of ϵ_s . The corresponding maximum percent error is 0.49 and the average percent error 0.11. As an additional check, the high salinity S -band data points were inverted to obtain ϵ_s . These values agreed with the polynomial fit to within the scatter of the L -band data.

The most obvious conclusion to be drawn from Figs. 2 and 3 is that the Stogryn regression produces values of ϵ_s that are consistently lower than the input data. This discrepancy increases with salinity, departing from the

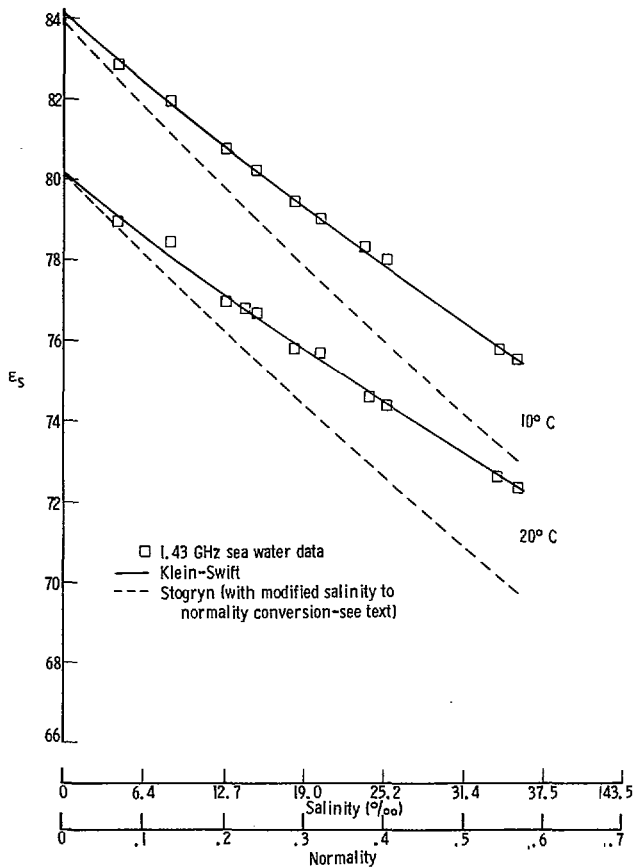


Fig. 3. Static dielectric constant of sea water as function of normality for temperatures of 10 and 20°C.

input data points by as much as 4 percent at a salinity of 36‰. Since Stogryn's results are based upon data utilizing NaCl solutions, Ho's experimental results for NaCl solutions were examined to determine whether or not the discrepancy is related to possible differences between the physical properties of sea water and sodium chloride solutions. Using Stogryn's expression for the conductivity of NaCl solutions, values of ϵ_s for 0.5N NaCl solutions were computed from Ho's data and then compared with the corresponding sea water values as derived from our polynomial fit. The two data sets agreed to within 1 percent over the entire temperature range of interest. We, therefore, conclude that the discrepancy is due to errors in the data that Stogryn used to develop his regression fit. In fact, the error appears to exceed the ± 1 and ± 2 unit accuracy claimed by Hasted and El-Sabeh [3] and Lane and Saxton [2], respectively.

The specific form of the polynomial fit through Ho's data is given by

$$\epsilon_s(T, S) = \epsilon_s(T) a(S, T) \quad (13)$$

where

$$\epsilon_s(T) = 87.134 - 1.949 \times 10^{-1} T - 1.276 \times 10^{-2} T^2 + 2.491 \times 10^{-4} T^3 \quad (14)$$

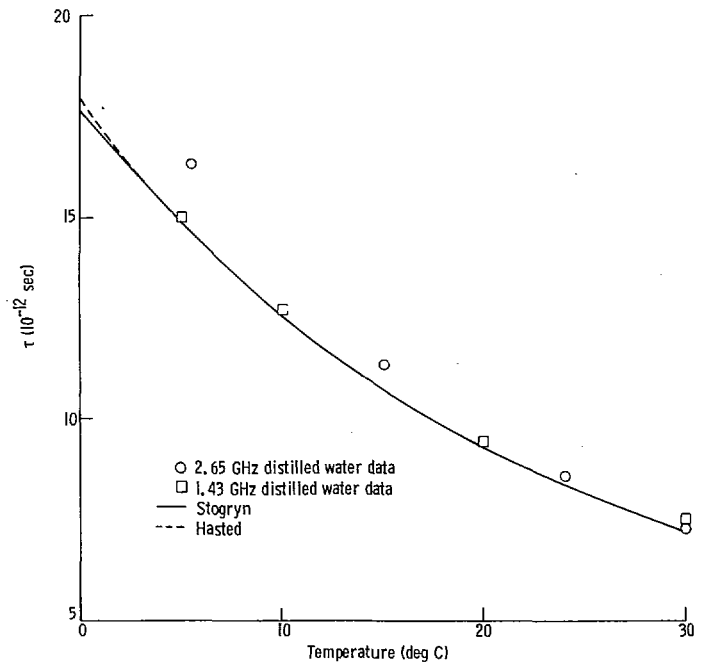


Fig. 4. Relaxation time of distilled water as function of temperature.

and

$$a(S, T) = 1.000 + 1.613 \times 10^{-5} ST - 3.656 \times 10^{-3} S + 3.210 \times 10^{-5} S^2 - 4.232 \times 10^{-7} S^3. \quad (15)$$

Computations derived from this expression will provide much more accurate values of the dielectric constant at the lower microwave frequencies. It should also be noted that the expression does not reduce to the distilled water expression (8) when $S = 0$ for reasons previously elaborated upon.

The application of Ho's results to calculations of relaxation time was much less satisfying. Equation (7) was used to calculate τ through the inversion of Ho's data. The results for distilled water are shown in Fig. 4. The solid curve is a plot of Stogryn's regression fit to the data of Grant *et al.* [5], and the dashed curve is a plot of the results of Hasted's [16] constraint fitting program. The L-band data points at 5, 10, 20, and 30°C are in excellent agreement with the published results; however, the values of τ as derived from the S-band results are too high at 5.5 and 15°C.

Since distilled water is used as the normalization in (1), the anomalous behavior of τ could have been caused by a poor S-band measurement of the real part, ϵ' , of the dielectric constant of distilled water. Therefore, ϵ' was recalculated at S-band using the Debye equation (5) with values of τ computed from the L-band data. The corrected value for ϵ' was then used to compute τ as a function of normality utilizing Ho's experimental data for 0.3, 0.4, 0.5, 0.6, and 0.7N NaCl solutions. The results are shown in Fig. 5 for temperatures of 5.5, 15, and 24°C along with corresponding results derived from Stogryn's regression equation. Ho's measured results are consistently higher than

TABLE V
HO AND HALL DATA USED IN DERIVING EXPRESSIONS FOR CONVERTING SALINITY INTO NORMALITY

NaCl Concn (W/V%)	3.30	3.28	3.21	3.20	3.15	3.14	3.12	3.07	2.94	2.78	1.48
Normality (W/V% / 58.46)	.564	.561	.549	.547	.539	.537	.534	.525	.503	.476	.253
Specific Gravity (From Cox ¹²)	1.0250	1.0249	1.0243	1.0242	1.0238	1.0238	1.0236	1.0232	1.0216	1.0209	1.0104
Chlorinity (W/V% X 6.0657 / Sp. Gr.)	19.53	19.41	19.01	18.95	18.66	18.60	18.49	18.20	17.46	16.52	8.88
Salinity (1.805 X chlorinity + .030)	35.28	35.07	34.34	34.23	33.71	33.60	33.40	32.88	31.55	29.85	16.06
Normality _g (Stogryn)	.617	.614	.601	.599	.589	.587	.584	.574	.551	.520	.277
Normality (Stogryn) Normality (W/V method)	1.0940	1.0945	1.0947	1.0951	1.0928	1.0931	1.0936	1.0933	1.0954	1.0924	1.0949

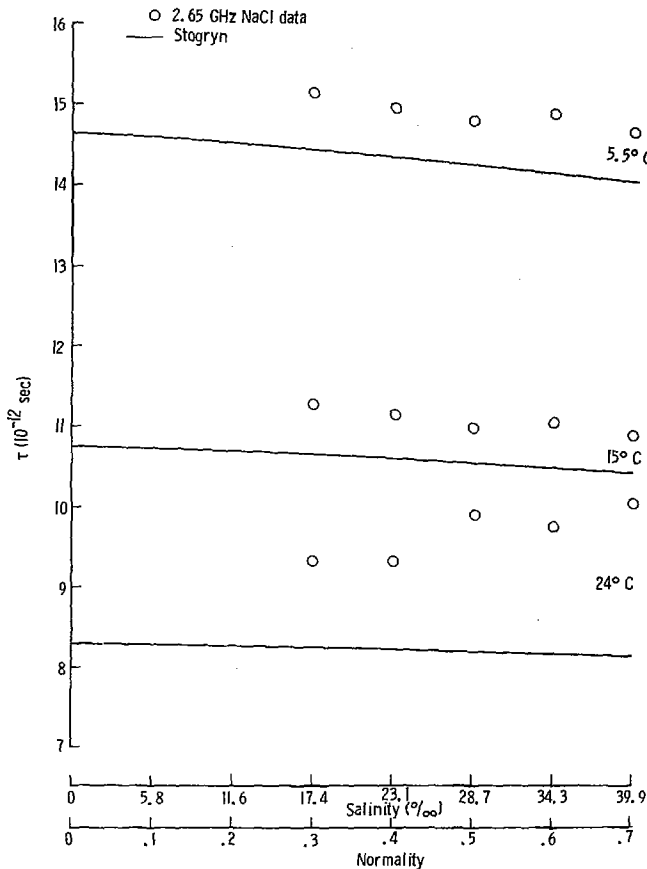


Fig. 5. Relaxation time of sodium chloride solutions as function of normality for temperatures of 5.5, 15, and 24°C.

the Stogryn values and substantially higher at 24°C for the higher salinity concentrations. Computations of τ were also undertaken using the raw data at 1.43 GHz. However, the scatter in τ was too large to be of value.

The trend of τ determined from Ho's data is particularly disturbing because prior experiments and theoretical results both indicate that τ decreases approximately linearly with increasing normality. Since the claimed accuracy of pre-

vious measurements [1]-[5] is $\pm 2.12 \times 10^{-13}$ s, we must conclude that Ho's measurements of ϵ'' suffer from systematic errors. Stogryn's expression for the relaxation time is, therefore, retained for the modeling of the dielectric constant and is given by

$$\tau(T,S) = \tau(T,0)b(S,T) \quad (16)$$

where $\tau(T,0)$ is the distilled water value of the relaxation time, whose regression equation is

$$\begin{aligned} \tau(T,0) = & 1.768 \times 10^{-11} - 6.086 \\ & \times 10^{-13}T + 1.104 \times 10^{-14}T^2 \\ & - 8.111 \times 10^{-17}T^3 \end{aligned} \quad (17)$$

and $b(S,T)$ has been modified, as explained below, to

$$\begin{aligned} b(S,T) = & 1.000 + 2.282 \times 10^{-5}ST - 7.638 \\ & \times 10^{-4}S - 7.760 \times 10^{-6}S^2 \\ & + 1.105 \times 10^{-8}S^3. \end{aligned} \quad (18)$$

As a side issue of interest, Stogryn expresses (16) in terms of normality instead of salinity and then gives a polynomial equation which converts salinity into normality. While his expression is valid for sodium chloride solutions, it was discovered that the normality as computed from the sodium chloride concentrations of the S-band sea water samples, as given in Table V, were smaller on the average by a factor of 1.0940 than that resulting from Stogryn's conversion equation. The correct relationship between the normality and salinity of sea water is given by

$$\begin{aligned} N = & 0.9141S(1.707 \times 10^{-2} + 1.205 \\ & \times 10^{-5}S + 4.058 \times 10^{-9}S^2). \end{aligned} \quad (19)$$

For sodium chloride solutions, the factor of 0.9141 is suppressed.

DISCUSSION OF ERRORS

The brightness temperature T_B of an emitter of microwave radiation is related to the molecular temperature of the source T through the emissivity e , such that

$$T_B = eT. \quad (20)$$

If the emitter fills a flat half-space, the emissivity, at normal incidence, is given by

$$e = 1 - \left| \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}} \right|^2 \quad (21)$$

where the second term in (21) is the Fresnel reflection coefficient and ϵ is the dielectric constant of the emitter. By differentiating (21), the following equation for the change in emissivity as a function of the change in dielectric constant is obtained:

$$\delta e = 2(1 - e) \operatorname{Re} \frac{\delta \epsilon}{\epsilon^{3/2}} \quad (22)$$

where it has been assumed that $\epsilon \gg 1$. Taking $\epsilon = 75 + j42$, which is the approximate value of the dielectric constant of sea water at 1.43 GHz when $S = 20\text{‰}$ and $T = 20^\circ\text{C}$, it then follows that the error associated with this particular choice is

$$\delta e \cong 1.15(\delta \epsilon' + \delta \epsilon'') \times 10^{-3}. \quad (23)$$

Using Ho's estimated error of 0.2 percent in ϵ' , and computing the error in ϵ'' from the assumed error of 2.12×10^{-13} s in τ using the Debye equation, it follows that $\delta \epsilon' \cong 0.15$ and $\delta \epsilon'' \cong 0.13$. Hence, the error in the brightness temperature with $T \cong 293$ K is

$$\delta T_B = \delta e T \cong 0.09 \text{ K} \quad (24)$$

which is well within the absolute accuracy of radiometers [6] currently used for remote sensing of the ocean. Fig. 6 contains a plot of brightness temperature versus molecular temperature of the sea, with salinity as a parameter. This shows that if the surface temperature is accurately known, then the salinity near $S = 20\text{‰}$ can be measured to an accuracy much better than 1‰ for the particular error given by (24). The curve also shows that it becomes very difficult to measure salinity for salinities lower than 5‰ .

Temperature can be remotely measured independently with an infrared radiometer; however, the infrared signature can be degraded by the atmosphere. The approach taken at the Langley Research Center is to use two precision radiometers with operating frequencies separated by approximately an octave. Since the radiometric signature is different at the two frequencies, an inverse regression can be developed to obtain temperature as well as salinity from the two measured brightness temperatures. Computer studies have shown that the inversion will give accuracies of at least 0.5 K for water temperature and 1‰ for salinity when $S > 5\text{‰}$. Major sources of error will occur from changes in the emissivity due to surface roughness, foam, and sun glint. However, these effects are beyond the scope of this paper.

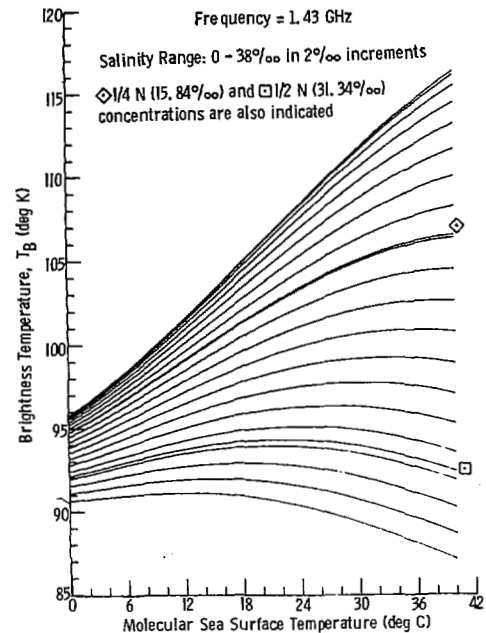


Fig. 6. Brightness temperature versus molecular temperature of smooth sea at normal incidence for various salinities at $f = 1.43$ GHz. Curves show effects of different salinities in increments of 2‰ , with 0‰ at top of figure and 38‰ at bottom. $\frac{1}{4}N$ (15.84‰) and $\frac{1}{2}N$ (31.34‰) concentrations are also indicated on figure.

Errors in τ and ϵ_∞ have a minor impact at the lower microwave frequencies because the product $\omega\tau$ is a small number. However, as the frequency increases to X -band, $\omega\tau \cong 1$ and the error in ϵ' is maximized. At X -band, the uncertainty in ϵ' is greater than one unit, and the target accuracy of 0.3 K in brightness temperature is marginal. At frequencies higher than X -band, the error in ϵ' due to uncertainties in τ rapidly diminishes as ω^{-2} and the dominant source of error is ϵ_∞ , as expected.

With the wealth of advanced microwave and millimeter wave components that are presently available, it would be highly desirable to measure τ and ϵ_∞ of sea water over a range of salinities and temperatures in order to confirm and possibly upgrade the quoted accuracies. This recommendation can be appreciated upon inverting the raw data for ϵ' and ϵ'' given in the early references, and observing the systematic variations and associated errors in τ at each frequency.

CONCLUDING REMARKS

An improved model for the dielectric constant of sea water has been developed which will yield an accuracy of at least 0.3 K in brightness temperature for undisturbed seas at frequencies below X -band. The results have shown that the standard Debye expression ($\alpha = 0$) provides an adequate description of the dielectric constant, and a choice of $\epsilon_\infty = 4.9 \pm 20$ percent contributes negligible error. A substantial improvement in the regression for ϵ_s was given and it was shown that a 4 percent error in τ is tolerable at the lower microwave frequencies. The computational model to be used for the dielectric constant of sea water is thus given by (5), where ϵ_s is described by (13)–(15), τ by (16)–(18),

σ by (9)–(12), and α and ϵ_∞ as above. These equations should be valid for salinities in the range 4–35‰ as these are the values from which the data used to develop ϵ_s were obtained. As pointed out in the last section, since it is difficult to remotely measure salinities below 5‰, the lower salinity limit should not be restrictive. In order to calculate the static dielectric constant of distilled water, however, (8) should be used.

Errors in τ and ϵ_∞ have an impact at the higher microwave frequencies. It is, therefore, recommended that a measurement program be undertaken to define these parameters more accurately.

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Microwave Radiometric Determination of Wind Speed at the Surface of the Ocean During BESEX

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Abstract—Microwave radiometric measurements were made at wavelengths ranging from 0.8 to 2.8 cm at altitudes from 0.16 to 11 km under well documented meteorological conditions over the Bering Sea. It is shown that determinations of wind speed at the ocean surface and liquid water content of the clouds may be made from such data. Determinations were made from two simultaneous but independent sets of radiometric measurements. The wind speeds and liquid water contents made from these two sets showed remarkable agreement. Independent estimates of these parameters made from *in situ* measurements showed reasonable agreement as well.

INTRODUCTION

THE BERING Sea Expedition (BESEX) was a joint venture between the U.S. and the U.S.S.R. primarily for the purpose of comparing and exchanging techniques of microwave remote sensing. The organization of the program and the joint results are discussed by Kondrat'yev et al. [1]. For the purposes of this paper we will consider only the data from the NASA CV-990 aircraft for March 2, 1973, for which the flight lines and meteorological conditions were suitable for demonstrating the possibility of measuring wind speeds at the surface of the ocean by means of microwave radiometry. To this end we will express brightness temperatures for various wavelengths and viewing geom-

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